Name _____ Date _____ Class _____

LESSON 10-3

Practice A

Formulas in Three Dimensions

Match the letter of each formula to its name.

1. Euler's Formula _____

2. diagonal of a rectangular prism _____

3. distance in three dimensions _____

4. midpoint in three dimensions _____

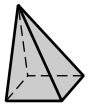
a. $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

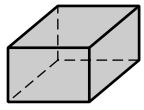
b. $V - E + F = 2$

c. $d = \sqrt{\ell^2 + w^2 + h^2}$

d. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Count the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's Formula.

5. 

6. 

_____ _____

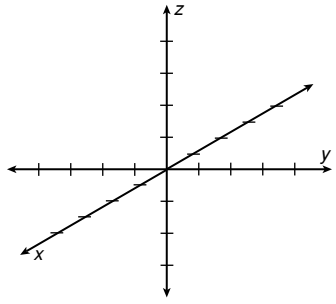
For Exercises 7–9, use the formula for the length of a diagonal to find the unknown dimension in each polyhedron. Round to the nearest tenth.

7. the length of a diagonal of a cube with edge length 3 in. _____

8. the length of a diagonal of a 7-cm-by-10-cm-by-4-cm rectangular prism _____

9. the height of a rectangular prism with a 6-m-by-6-m base and a 9 m diagonal _____

10. A rectangular prism with length 3, width 2, and height 4 has one vertex at (0, 0, 0). Three other vertices are at (3, 0, 0), (0, 2, 0), and (0, 0, 4). Find the four other vertices. Then graph the figure.



_____

Use the formula for distance in three dimensions to find the distance between the given points. Use the midpoint formula in three dimensions to find the midpoint of the segment with the given endpoints. Round to the nearest tenth if necessary.

11. (0, 0, 0) and (2, 4, 6)

12. (1, 0, 5) and (0, 4, 0)

_____ _____

13. The world's largest ball of twine wound by a single individual weighs 17,400 pounds and has a 12-foot diameter. Roman climbs on top of the ball for a picture. To take the best picture, Lysandra moves 15 feet back and then 6 feet to her right. Find the distance from Lysandra to Roman. Round to the nearest tenth. _____

LESSON 10-3

Practice A

Formulas in Three Dimensions

Match the letter of each formula to its name.

1. Euler's Formula — b
2. diagonal of a rectangular prism — c
3. distance in three dimensions — d
4. midpoint in three dimensions — a

a. $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

b. $V - E + F = 2$

c. $d = \sqrt{\ell^2 + w^2 + h^2}$

d. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Count the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's Formula.

5. 

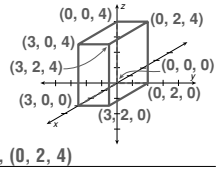
V = 5; *E* = 8; *F* = 5; 5 − 8 + 5 = 2

6. 

V = 8; *E* = 12; *F* = 6; 8 − 12 + 6 = 2

For Exercises 7–9, use the formula for the length of a diagonal to find the unknown dimension in each polyhedron. Round to the nearest tenth.

7. the length of a diagonal of a cube with edge length 3 in. — 5.2 in.
8. the length of a diagonal of a 7-cm-by-10-cm-by-4-cm rectangular prism — 12.8 cm
9. the height of a rectangular prism with a 6-m-by-6-m base and a 9 m diagonal — 3 m
10. A rectangular prism with length 3, width 2, and height 4 has one vertex at (0, 0, 0). Three other vertices are at (3, 0, 0), (0, 2, 0), and (0, 0, 4). Find the four other vertices. Then graph the figure.



(3, 2, 0), (3, 2, 4), (3, 0, 4), (0, 2, 4)

Use the formula for distance in three dimensions to find the distance between the given points. Use the midpoint formula in three dimensions to find the midpoint of the segment with the given endpoints. Round to the nearest tenth if necessary.

11. (0, 0, 0) and (2, 4, 6) — 7.5 units; (1, 2, 3)
12. (1, 0, 5) and (0, 4, 0) — 6.5 units; (0.5, 2, 2.5)
13. The world's largest ball of twine wound by a single individual weighs 17,400 pounds and has a 12-foot diameter. Roman climbs on top of the ball for a picture. To take the best picture, Lysandra moves 15 feet back and then 6 feet to her right. Find the distance from Lysandra to Roman. Round to the nearest tenth. — 24.9 feet

LESSON 10-3

Practice B

Formulas in Three Dimensions

Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's Formula.

1. 

V = 6; *E* = 12; *F* = 8; 6 − 12 + 8 = 2

2. 

V = 7; *E* = 12; *F* = 7; 7 − 12 + 7 = 2

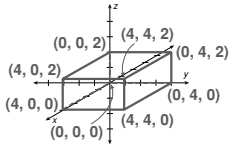
Find the unknown dimension in each polyhedron. Round to the nearest tenth.

3. the edge length of a cube with a diagonal of 9 ft — 5.2 ft
4. the length of a diagonal of a 15-mm-by-20-mm-by-8-mm rectangular prism — 26.2 mm
5. the length of a rectangular prism with width 2 in., height 18 in., and a 21-in. diagonal — 10.6 in.

Graph each figure.

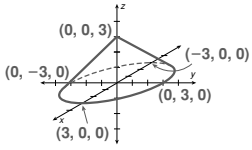
6. a square prism with base edge length 4 units, height 2 units, and one vertex at (0, 0, 0)

Possible answer:



7. a cone with base diameter 6 units, height 3 units, and base centered at (0, 0, 0)

Possible answer:



Find the distance between the given points. Find the midpoint of the segment with the given endpoints. Round to the nearest tenth if necessary.

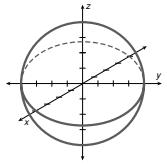
8. (1, 10, 3) and (5, 5, 5) — 6.7 units; (3, 7.5, 4)
9. (−8, 0, 11) and (2, −6, −17) — 30.3 units; (−3, −3, −3)

LESSON 10-3

Practice C

Formulas in Three Dimensions

1. The distance from (0, 0, 0) to the surface of a solid is 4 units. Graph the solid.



2. Each edge of the solid shown in the figure measures 5 in. Find the length of $\overline{AB}$. Give an exact answer and an answer rounded to the nearest tenth.

$5\sqrt{2}$ in.; 7.1 in.

3. Find the length of $\overline{AB}$ if the bipyramid in Exercise 2 were based on a triangle rather than on a square. Round to the nearest tenth. — 8.2 in.
4. Find the length of $\overline{AB}$ if the bipyramid in Exercise 2 were based on a pentagon rather than on a square. Round to the nearest tenth. — 5.3 in.
5. If the bipyramid in Exercise 2 were based on a hexagon instead of a square, describe what sort of shape would result. Explain your answer.

The shape would be a flat hexagon; possible answer: The distance to the vertex of the bipyramid from the midpoint of a side (the slant height) would be $\frac{5\sqrt{3}}{2}$ in. The distance from the midpoint of a side to the center of the hexagon (the apothem) would also be $\frac{5\sqrt{3}}{2}$ in. Therefore, the height *AB* would be zero.

6. The distance from *A*(−2, 7, 0) to *B*(3, 2, *b*) and from *A* to *C*(3, 2, *c*) is 10 units. *D* lies on $\overline{BC}$ so that $\overline{AD}$ is the shortest distance from *A* to $\overline{BC}$. Find the coordinates of *D* without calculating. Explain how you got the answer.

D(3, 2, 0); possible answer: Because *B* and *C* have the same *x*- and *y*-coordinates, *D* must also have those *x*- and *y*-coordinates to lie on $\overline{BC}$. Any difference in length from *A* to $\overline{BC}$ is caused by changes in the *z*-coordinate, and the shortest distance occurs when *D* has the same *z*-coordinate as *A*.

7. A rectangular prism has vertices, in no particular order, at (−10, 8, 2), (−15, 8, 10), (−10, 5, 10), (−10, 5, 2), (−10, 8, 10), (−15, 5, 2), (−15, 5, 10), and (−15, 8, 2). Find the length of a diagonal of the prism. Round to the nearest tenth. — $7\sqrt{2} \approx 9.9$ units
8. Find the coordinates of a point that is equidistant from each of the eight vertices of the prism in Exercise 7. — (−12.5, 6.5, 6)

Tyrone has eight 1-in. cubes. He arranges all eight of them to make different rectangular prisms. Find the dimensions of the prisms based on the diagonal lengths given below.

9. $\sqrt{66}$ in. — an 8-by-1-by-1 prism
10. $\sqrt{21}$ in. — a 4-by-2-by-1 prism
11. $2\sqrt{3}$ in. — a 2-by-2-by-2 prism

LESSON 10-3

Reteach

Formulas in Three Dimensions

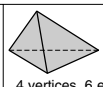
A **polyhedron** is a solid formed by four or more polygons that intersect only at their edges. Prisms and pyramids are polyhedrons. Cylinders and cones are not.

Euler's Formula

For any polyhedron with *V* vertices, *E* edges, and *F* faces,

$V - E + F = 2$.

Example



$V - E + F = 2$
$4 - 6 + 4 = 2$
$2 = 2$

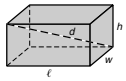
4 vertices, 6 edges, 4 faces

Euler's Formula: $V = 4$, $E = 6$, $F = 4$

Diagonal of a Right Rectangular Prism

The length of a diagonal *d* of a right rectangular prism with length ℓ, width *w*, and height *h* is

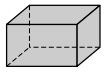
$d = \sqrt{\ell^2 + w^2 + h^2}$.



Find the height of a rectangular prism with a 4 cm by 3 cm base and a 7 cm diagonal.

$d = \sqrt{\ell^2 + w^2 + h^2}$	Formula for the diagonal of a right rectangular prism
$7 = \sqrt{4^2 + 3^2 + h^2}$	Substitute 7 for *d*, 4 for ℓ, and 3 for *w*.
$49 = 4^2 + 3^2 + h^2$	Square both sides of the equation.
$24 = h^2$	Simplify.
$4.9 \text{ cm} \approx h$	Take the square root of each side.

Find the number of vertices, edges, and faces of each polyhedron. Use your results to verify Euler's Formula.

1. 

vertices: 8; edges: 12; faces: 6; 8 − 12 + 6 = 2

2. 

vertices: 6; edges: 10; faces: 6; 6 − 10 + 6 = 2

Find the unknown dimension in each figure. Round to the nearest tenth if necessary.

3. the length of the diagonal of a 6 cm by 8 cm by 11 cm rectangular prism — $d \approx 14.9$ cm
4. the height of a rectangular prism with a 4 in. by 5 in. base and a 9 in. diagonal — $h \approx 6.3$ in.